Microalgae Activated Sludge: Process Modeling and Optimization

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Abstract

This work deals with steady-state simulation study of a process formed by a microalgae-bacteria photobioreactor (PBR) in an activated sludge configuration. In particular, the process behavior in terms of variations in the sludge retention time and carbon dioxide (CO_2) injected is presented. The optimization is done by considering the total PBR volume as two volumes in series, and aiming for the minimal substrate concentration in the effluent, for a given external light and CO_2 injected. Results suggest that it is possible to obtain an optimum volume distribution of the process that gives a lower effluent substrate concentration compared to the same process using a single volume.

Keywords: microalgae-bacteria, bioprocess design, effluent minimization, photobioreactor, volume distribution.

1 Background

In wastewater treatment applications, the bioreactors are disposed in a configuration known as activated sludge process (ASP), where the bioreactor effluent is connected to a setter. The settler increases the microorganism (biomass) concentration and part of the settled stream is recycle back to the bioreactor (Grady Jr. et al., 1999). The biomass separation given by the settler makes the residence time of particulate components greater than the residence time of soluble components. This residence time for particulate components, referred as the sludge retention time (SRT), is a key factor in the plant operation. SRT is defined as the ratio between the amount of biomass in the bioreactor and the amount of removed biomass per time unit, i.e. it represents the average time the biomass stays in a bioreactor. External aeration is another key factor in these processes, since it is needed for aerobic bacterias to consume nutrients (such as nitrogen and phosphorus).

Nowadays, the role of microalgae in wastewater treatment applications is becoming more relevant (de la Noüe et al., 1992; Dalrymple et al., 2013). Via photosynthesis, where a certain external illumination is applied to the bioreactor, microalgae require carbon dioxide to consume nitrogen and release oxygen, which is beneficial for the aerobic bacterias in the wastewater treatment processes. In this way, we refer to photobioreactors (PBRs) as bioreactors able to grow microalgae.

In a PBR the biological dynamics is directly affected by the irradiance applied. Therefore, several models for the biology and the irradiance have been proposed in literature. Concerning the biological models, early works can be found in Droop (1968, 1973), where the growth rate of the microalgae is assumed to be associated to an internal substrate concentration. The basic form of this model includes three ordinary differential equations which describe the substrate (nutrient), the microalgae and the internal substrate cell quota in the microalgae. Jang and Baglama (2005) proposed a model which includes one main substrate and two microalgae species (zooplankton and phytoplankton). The study includes a global asymptotic analysis of the system considering different growth rates and changes in the input nutrient concentration. Decostere et al. (2013) proposed a model for microalgae growth on inorganic carbon which includes oxygen production. The model is based on the Activated Sludge Models (ASMs) and includes a calibration using data from respirometric-titrimetric experiments.

Regarding the models for the irradiance, several approaches have been proposed in the last decades. For example, Eilers and Peeters (1988) proposed a dynamic model for the irradiance, which links the light intensity and the rate of photosynthesis in phytoplankton microalgae. The model is based on physiological mechanisms, and includes the photoinhibition effect and the recovery from the photoinhibition. Geider et al. (1998) presented a simple model where the chlorophyll (a concentration that depends on the incident irradiance to the PBR) is included as a single variable. This models also includes the response of the photosynthesis to the nitrogen and light status in the microalgae. Other models linking the chlorophyll with the nutrient dynamics have been proposed, see for example Pahlow (2005). Results from this model replicate the nutrient:carbon ratio from experimental data.

The modeling of the microalgae-bacteria consortium has also been investigated in the last year. Dochain et al. (2003) reported a dynamic model with three microorganisms: microalgae, aerobic bacteria and sulphate-reducing anaerobic bacteria. The study includes a model calibration based on experimental data from different seasons. Zambrano et al. (2016) proposed a dynamic model for the microalgae-bacteria interaction, where the bacteria dynamics is inspired by the Activated

Sludge Model no. 1 (ASM1) (Henze et al., 1987) and the microalgae dynamics is inspired by the works from (Reichert et al., 2001) and (Solimeno et al., 2015). Experimental data from batch experiments presented by Krustok et al. (2016) was used for the model calibration.

When modeling bioreactors, a natural aim is to optimize the process in terms of volume and performance. The optimization of bioreactors has been investigated during decades (Aris, 1961; Herbert, 1964; Abu-Reesh, 1996), where mainly two typical approaches are done: (i) minimize the total bioreactor volume to achieve a given effluent substrate concentration, or (ii) from a given total volume of a set of bioreactors in series, optimize the volume distribution so to minimize the effluent substrate concentration.

The aim of the present study is, given a model for the microalgae-bacteria consortium, to study the behavior of a PBR-based activated sludge configuration (henceforth referred to as MAAS process) in terms of variations in key parameters such as the SRT and the CO_2 injected. The study includes the optimization of the total volume distribution when two PBRs in series are considered, so to minimize the effluent substrate concentration. A model based on Zambrano et al. (2016) is used for describing the microalgae-bacteria consortium. This model includes a modification in the effect that the irradiance has on the biological activity, which now depends on the amount of microalgae and bacteria concentration.

The paper is organized as follows. A description of the biological process and the model are given in Section 2. Section 3 gives a numerical illustration, and some conclusions are given in Section 4.

2 Methods

2.1 The MAAS process

The configuration of the MAAS process is shown in Figure 1, which consists of a PBR and a clarifier. In the PBR, the wastewater is treated by the biological activity of the microalgae-bacteria consortium. An external illumination and CO_2 injection is applied to the PBR. In the clarifier (also called settler), the microorganisms are separated from the treated water. The PBR and clarifier are interconnected following the classical configuration of an ASP (Grady Jr. et al., 1999). To maintain the biomass population, part of the underflow from the clarifier goes as return sludge back to the PBR and the excess sludge is removed.

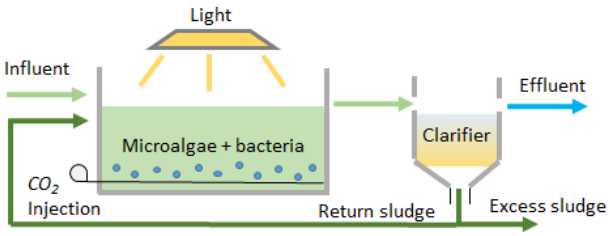

Figure 1. Layout of the MAAS process.

Since the main purpose of this study was to analyze the overall behavior of the MAAS process, an ideal clarifier was assumed, i.e, the amount of solids in the effluent is neglected, which means that all the sludge is thickened.

2.2 The model

A simple model for the microalgae-bacteria interaction (Zambrano et al., 2016) was used to describe the biological activity in the MAAS process. The model is formed by six components: two main biomass populations (microalgae (X_{alg}) and bacteria (X_{bac})), two dissolved substrate concentrations (ammonium (S_{nh4}) and nitrate (S_{no3})), and two dissolved gases concentrations (oxygen (S_{o2}) and carbon dioxide (S_{co2})), see Table 1.

The model is based on the following assumptions. There is only one class of microalgae and one class of bacteria. The microalgae growth on dissolved ammonium and nitrate and it is assumed that ammonium is preferred (Reichert et al., 2001). The autotrophic conversion of ammonium by the bacteria is considered as a single step process with the aid of oxygen (Henze et al., 1987).

Table 1. Model components and stoichiometric matrix.

Component $(i) \rightarrow$	(1)	(2)	(3)	(4)	(5)	(6)	Process rate
Process $(j) \downarrow$	X_{alg} $\left[\frac{\text{gCOD}}{\text{m}^3}\right]$	X_{bac} $\left[\frac{\text{gCOD}}{\text{m}^3}\right]$	S_{nh4} $\left[\frac{\text{gN}}{\text{m}^3}\right]$	S_{no3} $\left[\frac{\text{gN}}{\text{m}^3}\right]$	S_{o2} $\left[\frac{\text{gO}_2}{\text{m}^3}\right]$	S_{co2} $\left[\frac{\text{gCO}_2}{\text{m}^3}\right]$	ρ_j $\left[\frac{\text{g}}{\text{m}^3\text{d}}\right]$
(1) Algae growth on NH_4	1		$-\frac{1}{Y^{\text{N}}_{\text{alg,nh4}}}$		$Y^{\text{O}}_{\text{alg,nh4}}$	$-\frac{1}{Y^{\text{C}}_{\text{alg,nh4}}}$	ρ_1
(2) Algae growth on NO_3	1			$-\frac{1}{Y^{\text{N}}_{\text{alg,no3}}}$	$Y^{\text{O}}_{\text{alg,no3}}$	$-\frac{1}{Y^{\text{C}}_{\text{alg,no3}}}$	ρ_2
(3) Algae decay	-1		$f^{\text{N}}_{\text{alg}}$			$f^{\text{C}}_{\text{alg}}$	ρ_3
(4) Bacteria growth		1	$-i_{\text{X}_{\text{bac}}} - \frac{1}{Y_{\text{bac}}}$	$\frac{1}{Y_{\text{bac}}}$	$-\left(\frac{4.57-Y_{\text{bac}}}{Y_{\text{bac}}}\right)$	$f^{\text{C}}_{\text{bac}}$	ρ_4
(5) Bacteria decay		-1	$i_{\text{X}_{\text{bac}}}$				ρ_5

The dependency of stoichiometric and biokinetics factors on temperature was not included. The inhibition of microalgae by excess of light or excess of CO_2 was not considered. The different processes and the stoichiometry involved in the biological model are shown in Table 1, whereas Table 2 shows the correspondent expressions for the process rates (ρ).

Table 2. Process rates.

ρ_j	Process rate
ρ_1	$\mu_{\text{alg}}\mu(I)\left(\frac{S_{\text{co2}}}{K_{\text{co2}}+S_{\text{co2}}}\right)\left(\frac{S_{\text{nh4}}}{K_{\text{n,alg}}+S_{\text{nh4}}}\right)X_{\text{alg}}$
ρ_2	$\mu_{\text{alg}}\mu(I)\left(\frac{S_{\text{co2}}}{K_{\text{co2}}+S_{\text{co2}}}\right)\left(\frac{S_{\text{no3}}}{K_{\text{n,alg}}+S_{\text{no3}}}\right)\left(\frac{K_{\text{n,alg}}}{K_{\text{n,alg}}+S_{\text{nh4}}}\right)X_{\text{alg}}$
ρ_3	$b_{\text{alg}}X_{\text{alg}}$
ρ_4	$\mu_{\text{bac}}\left(\frac{S_{\text{nh4}}}{K_{\text{n,bac}}+S_{\text{nh4}}}\right)\left(\frac{S_{\text{o2}}}{K_{\text{o2}}+S_{\text{o2}}}\right)X_{\text{bac}}$
ρ_5	$b_{\text{bac}}X_{\text{bac}}$

where $\mu(I) = I/(K_{\text{I}}+I)$.

In this work, a modification in the model for the irradiance was introduced. In Zambrano et al. (2016), the model for the irradiance considers that the illumination applied to the PBR does not change under any circumstances when it travels through the reactor, i.e. the irradiance I is constant in Table 2. Now, the model includes the effect of the biomass concentration on the light penetration. This was done in a similar way as the Beer-Lambert law (Huisman et al., 2002), giving the following irradiance factor:

$$\mu(\tilde{I}) = \frac{\tilde{I}(X_{\text{alg}}, X_{\text{bac}})}{K_I + \tilde{I}(X_{\text{alg}}, X_{\text{bac}})}, \quad (1)$$

$$\text{where} \quad \tilde{I}(X_{\text{alg}}, X_{\text{bac}}) = I \times \exp\left[-\alpha(X_{\text{alg}} + X_{\text{bac}})\right], \quad (2)$$

where I [μmol/m^{2}s] is the total irradiance applied to the PBR, K_I [μmol/m^{2}s] is a half-saturation constant, and α [m^3/g] is the specific light attenuation coefficient. Expression (1) replaces $\mu(I)$ in the process rates for the algae growth on ammonium and nitrate (cf. Table 2). The rest of the model parameters are described in Table 3. See the reference of the parameters in Zambrano et al. (2016).

Since the PBR is assumed to be a completely mixed tank reactor, the expression (1) considers a homogeneous concentration of biomass in the liquid, therefore not dependency with depth was included. For simplicity, it is assumed that both microalgae and bacteria interrupt the light in the same way.

The combined effect of water-atmosphere gas exchange and gas injection were modeled as separated processes. Both processes follow the well known mass-transfer model:

$$G_{tr,gas} = K_L a_{gas}(S^{sat}_{gas} - S_{gas}), \quad (3)$$

Table 3. Model parameters.

Symbol	Definition [Unit]	Value
b_{alg}	Algae decay [1/d]	0.1
b_{bac}	Bacteria decay [1/d]	0.05
$f^{\text{C}}_{\text{bac}}$	CO_2 produced per bacteria [gCO_2/gCOD]	1.375
$f^{\text{C}}_{\text{alg}}$	CO_2 fraction in algae [gCO_2/gCOD]	0.383
$f^{\text{N}}_{\text{alg}}$	N fraction in algae [gN/gCOD]	0.065
I	Irradiance [μmol/m^{2}s]	100
$i_{\text{X}_{\text{bac}}}$	N used in bacteria growth [gN/gCOD]	0.08
$K_L a_{\text{o2}}$	Mass transfer coeff. O_2 [1/d]	4
$K_L a_{\text{co2}}$	Mass transfer coeff. CO_2 [1/d]	3.538
$K_L a_{\text{co}_2,\text{inj}}$	Mass transfer coeff. CO_2 injected [1/d]	0-2.5
K_{co2}	Algae half-sat. coeff. for C [gC/m^3]	4×10^{-3}
K_{I}	Algae half-sat. coeff. for I [μmol/m^{2}s]	25
$K_{\text{n,alg}}$	Algae half-sat. coeff. for N [gN/m^3]	0.1
$K_{\text{n,bac}}$	Bacteria half-sat. coeff. for N [gN/m^3]	1
K_{o2}	Bacteria half-sat. coeff. for O_2 [gO_2/m^3]	0.4
$S^{\text{sat}}_{\text{o2}}$	Sat. concentration for O_2 in water [gO_2/m^3]	8.32
$S^{\text{sat}}_{\text{co2}}$	Sat. concentration for CO_2 in water [gCO_2/m^3]	0.546
Y_{bac}	Bacteria growth yield [gCOD/gN]	0.24
$Y^{\text{C}}_{\text{alg,nh4}}$	Algae CO_2 yield on NH_4 [gCOD/gCO_2]	0.842
$Y^{\text{N}}_{\text{alg,nh4}}$	Algae N yield on NH_4 [gCOD/gN]	11.91
$Y^{\text{O}}_{\text{alg,nh4}}$	Algae O_2 yield on NH_4 [gO_2/gCOD]	0.996
$Y^{\text{C}}_{\text{alg,no3}}$	Algae CO_2 yield on NO_3 [gCOD/gCO_2]	0.622
$Y^{\text{N}}_{\text{alg,no3}}$	Algae N yield on NO_3 [gCOD/gN]	3.415
$Y^{\text{O}}_{\text{alg,no3}}$	Algae O_2 yield on NO_3 [gO_2/gCOD]	1.301
α	Light attenuation coefficient [m^3/g]	5×10^{-4}
μ_{alg}	Algae specific growth rate [1/d]	1.6
μ_{bac}	Bacteria specific growth rate [1/d]	0.5

where $G_{tr,gas}$ is the amount of gas transfered from/to the atmosphere, $K_L a_{gas}$ is the mass transfer coefficient between the gas and the liquid phase, S^{sat}_{gas} is the saturation concentration of the gas, and S_{gas} is the dissolved gas concentration.

3 Results and Discussions

This section shows a numerical example of the MAAS process. Two main cases were evaluated: PBR as a single volume and as two volumes in series. The process was evaluated in steady-state conditions for different values of SRT and CO_2 injected. The SRT was adjusted by modifying the amount of excess sludge from the process.

The model programming and the simulation results

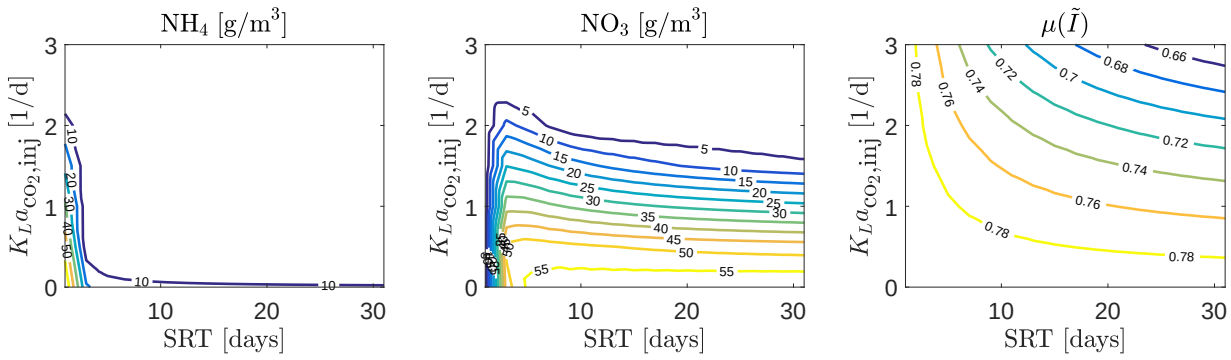

Figure 2. MAAS process with a single PBR. Contour plots showing steady-state values of NH_4 and NO_3 in the effluent, and $\mu(\tilde{I})$ as a function of SRT and $K_L a_{co_2,inj}$. $K_L a_{co_2,inj}$ refers to mass transfer coefficient of CO_2 injected. $\alpha = 5 \times 10^{-4}$ m^3/g.

were obtained using the MATLAB®/Simulink platform.

3.1 Influent characteristics and process parameters

We consider a PBR with total volume of $V = 0.07$ m^3 and influent flow rate of 10.8 L/d. A constant influent flow rate was applied, with a composition of 70 g/m^3 of dissolved NH_4, 2 g/m^3 of dissolved NO_3, and no biomass concentration. The value of the model parameters used for the simulations are described in Table 3.

3.2 MAAS process with one PBR

The MAAS process was first simulated with a single PBR of volume V. The effluent ammonium concentration (NH_4), nitrate concentration (NO_3) and irradiance factor (cf. Expression (1)) were evaluated for different SRTs and CO_2 injected. Results are shown as contour plots in Figure 2, where a light attenuation coefficient $\alpha = 5 \times 10^{-4}$ m^3/g was used.

See that the effluent NH_4 concentration is almost consumed in a wide range of SRT and that a low injection of CO_2 is needed. This is not the case for the effluent NO_3, where high NO_3 concentration is obtained when low CO_2 is applied. See that this concentration decreases as the injected CO_2 increases, this is expected since the microalgae is the only microorganism that can consume this substrate (by injecting CO_2). Also note that the effluent NO_3 concentration does not show a significant change under variations in the SRT. As expected, for very low values in the SRT, the effluent NH_4 concentration starts to increase towards values of the influent concentration, i.e. the process is very close to wash-out condition.

Note also in Figure 2 that the irradiance factor decreases when SRT or the CO_2 injected increase. An increasing in the SRT promotes an accumulation of microalgae and bacteria concentration in the PBR, and more CO_2 injected promotes an increasing in the microalgae concentration. Therefore, this increment in the microorganism concentration results in a decreasing of the irradiance factor (cf. Expressions (1)-(2)).

3.3 MAAS process with two PBRs

Next, the process was simulated considering the entire volume as two PBRs in series, subject to the restriction $V = V_1 + V_2$. It was decided that the irradiance applied to each PBR was proportional to its volume. Therefore, from the total irradiance I used in the case of a single PBR, now we have:

$$I_1 = \frac{V_1}{V} I, \quad I_2 = I - I_1, \tag{4}$$

where I_1 and I_2 are the irradiance in volumes V_1 and V_2 respectively.

From Figure 2, several points were taken as operational point to be optimized by distributing the total PBR volume V into two PBRs in series. As illustration, points with SRT = 15 d were selected. Different values for V_1 and CO_2 injected were evaluated. The CO_2 injected was assumed to be the same for each PBR, results are shown in Figure 3(left). See that each curve has a certain optimum value for the first PBR volume V_1 when a maximum reduction in the NO_3 is achieved. Also note that this optimum V_1 decreases as the CO_2 injected increases. Zambrano and Carlsson (2014) reported a similar behavior for the case of optimizing several bioreactors in series in an activated sludge process, where a simple bioreactor model (one main microorganism and one main dissolved substrate) and a Monod function for describing the growth kinetics were used. See also in Figure 3(left) that there is a wide range of optimum V_1 when a large amount of CO_2 is injected.

Figure 3(right) shows the value of the irradiance factor in each PBR for different values of V_1 and CO_2 injected. Note that a low value in V_1 means a low value in the irradiance of this PBR (cf. Expression (4)). Therefore, the irradiance factor $\mu(\tilde{I}_1)$ is close to zero and this value increases as V_1 increases. Since the total volume is fixed, the situation is the opposite for V_2 and $\mu(\tilde{I}_2)$, i.e. when V_1 increases V_2 decreases. See also that for a given V_1,

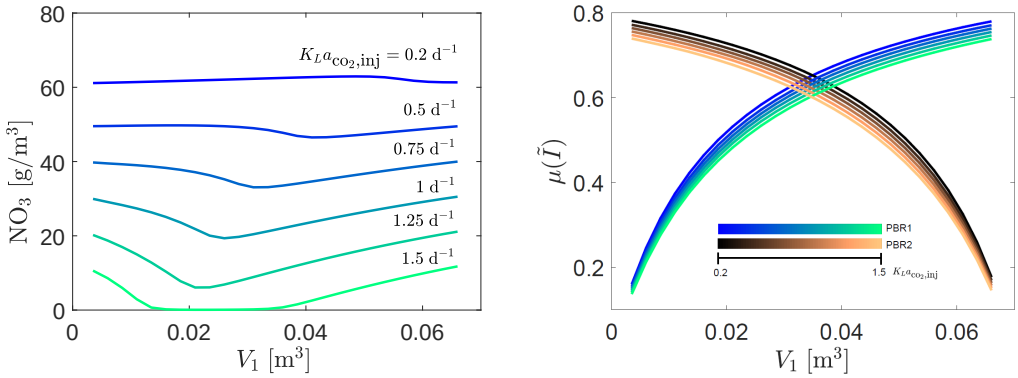

Figure 3. MAAS process with two PBRs in series. Left: Steady-state solution of effluent NO_3 as a function of V_1 and $K_La_{co_2,inj}$. Right: Irradiance factor for the first (PBR1) and second (PBR2) PBR, as a function of V_1 and $K_La_{co_2,inj}$. $\alpha = 5 \times 10^{-4}$ m^3/g.

the irradiance in each PBR decreases as the amount of CO_2 injected increases. As observed in Figure 2 for $\mu(\tilde{I})$ in a single PBR, this is because an increasing in the CO_2 promotes an increasing in the microalgae concentration, which reduces the amount of light penetration in the PBRs.

4 Conclusions

In this work, a steady-state simulation study of a PBR working in an ASP configuration was presented, referred as MAAS process. A simple model is used for the PBR, which includes one microalgae and bacteria species, two dissolved substrates and two dissolved gases. The model for the irradiation includes the effect of the microalgae and bacteria concentration in the PBR. This simple model gives relevant information about the behavior of the system for different SRTs and CO_2 injected.

Results show that, for a given SRT, it is possible to reduce the effluent substrate concentration by increasing the CO_2 injected, and this reduction is more sensitive to changes in the CO_2 injected than to changes in the SRT of the process. For the case of two PBRs in series, for a given SRT an optimum volume distribution can be achieved which depends on the CO_2 injected. This configuration gives a lower effluent substrate concentration than when a single PBR volume is assumed. Similar to the case of an ASP with bioreactors in series, one would expect that an increasing in the number of PBRs in series in the MAAS process would decrease the effluent substrate concentration.

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