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Steady-state analysis and design of activated sludge processes including compressive settling

Diehl, S.*, Zambrano, J. and Carlsson, B.****
* Centre for Mathematical Sciences, Lund University, P.O. Box 118, 22100 Lund, Sweden. (E-mail: diehl@maths.lth.se)
** Department of Information Technology, Uppsala University, P.O. Box 337, 75105 Uppsala, Sweden. (E-mail: jesus.zambrano@it.uu.se, bengt.carlsson@it.uu.se)

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Summary of key findings

A simplified activated sludge process (ASP) with one substrate and one particulate biomass component has been analyzed with respect to its steady states. The ASP consists of a completely stirred biological reactor and a settler. The biomass growths according to the Monod function and the decay rate is constant. In the considered one-dimensional settler model, the biomass undergoes hindered settling and compression according to constitutive functions that can be chosen by the investigator. The dynamic settler model is a nonlinear partial differential equation (PDE), so the steady-state equation is an ordinary differential equation. This is a difficulty in the steady-state analysis of an ASP. The key findings are the following:

- By analyzing the model PDE, a simple algebraic equation has been obtained that captures the necessary information related to the limiting flux of a settler in steady state in normal operation with a sludge blanket in the thickening zone. The equation relates the recycle concentration to the bulk velocity in the thickening zone with the sludge blanket kept at a specified level.
- All steady states of an ASP working in normal operation are given as a one-parameter family of solutions where the parameter is the recycle ratio r. Numerical solutions show that the effluent substrate concentration S^* is reduced significantly already for small r and is a decreasing function for all $r > 0$. The sludge age θ can be expressed as an explicit decreasing function of S^*.
- A control curve $w = w(r)$, relating the wastage ratio w to the recycle ratio r, gives the possible values that yield the desired steady state with the sludge blanket kept at a specified level.
- For given inputs (substrate concentration S_{in} and volumetric flow Q) to the ASP, a procedure is suggested for the design of an ASP that is constrained to work in normal operation. The procedure gives support in the decision on how to balance the building or start-up costs (which increase with the total area) and operational costs (which increase with r – energy for pumping water).

Background and relevance

Reduced models can capture the general behaviour of the ASP, its possibilities to reduce the dominant substrate concentration and the possibilities for controlling this by the pumps for the recycle and wastage streams. All this can be investigated by a steady-state analysis based on mass balances. Since the success and efficiency of an ASP depend on the volume of the bioreactor and the cross-sectional area of the settler, a related issue is the design of an ASP.

Finding general equations for the steady states of an ASP means that simplified assumptions and reduction in the number of equations have to be made. By taking only one particulate and one substrate component into account, one hopes to capture the behaviour of the most dominant components of an ASP. Although one can find such approaches in publications (Ajbar and AlZeghayer, 2013; Alqahtani et al., 2013; Kumar et al., 2009; Nelson et al., 2012), they contain very simplified assumptions, such as the settler behaves normally irrespective of loading conditions. Such an assumption may, however, not be satisfied in practice. The clarification-thickening process in the settler can be modelled with a nonlinear PDE. However, it is not straightforward to describe the steady-state solutions of such a PDE for a steady-state analysis of the entire ASP. Analyses on stability and possibilities for control of an ASP when only hindered settling is taken into account have been presented by Diehl and Farås (2012, 2013).

In addition to hindered settling, compression at high concentrations below the sludge blanket has a great influence on the capacity of the settler, which is related to the concept of limiting flux. This is included in the present work. To the best of the authors' knowledge this approach is novel and it will give the possibility for a more realistic design of an ASP than with previous methods.

Methods

The considered ASP consists of a bioreactor of volume V followed by a settler, from which there is a recycle flow to the reactor. The bioreactor is modelled as a completely stirred tank reactor and the settler as a one-dimensional tank in which hindered settling and compression take part. The influent volumetric flow rate Q to the ASP contains substrate dissolved in water at a concentration S_{in}. It is assumed that no biomass is present in the influent ($X_{\mathrm{in}} = 0$). The bioreactor output concentrations, which are equal to the feed concentration to the settler, are denoted by S^* and X^*. It is assumed that there are no reactions in the settler. The substrate concentration is thus unchanged and equal to S^* through the settler. The biomass effluent concentration is X_{e} and the recycle sludge concentration X_{r}. The recycle volumetric flow is rQ and the wastage flow wQ, where r and w are dimensionless control parameters. As for the kinetics in the bioreactor, a common growth rate of the biomass is the one by Monod:

$$\mu(S) := \mu_{\max} \frac{S}{K_{\mathrm{s}} + S},$$

where $\mu_{\max}$ is the maximum specific growth rate and K_{s} the half-saturation constant. The biomass is assumed to decay at the constant specific rate b. The processes in the settler are described by a PDE, which includes constitutive functions (which can be chosen by the investigator) for the hindered settling velocity and compression at high concentrations (Bürger et al., 2005, 2011).

The steady-state equations for the ASP consist of a couple of algebraic mass balances together with the steady-state equation of the settler PDE. By investigating the steady-state solutions and requiring that the settler should work in normal operation with a sludge blanket in the thickening zone, one can conclude that the sludge blanket level z_{sb} depends on X_{r} and the bulk velocity of the suspension in the thickening zone, which is (A_{S} is the settler cross-sectional area)

$$q = q(r, w) := \frac{Q(r + w)}{A_{\mathrm{S}}}, \tag{1}$$

see Figure 1. The right plot shows the contours along which $z_{\mathrm{sb}} = 0.5, \ldots, 3$ m. The total depth of the thickening zone is 3 m. A contour for a given z_{sb} is approximated by the following simple relation:

$$X_{\mathrm{r}} = U_{z_{\mathrm{sb}}}(q) := X_{z_{\mathrm{sb}}}^{\infty} \left(1 + \frac{\hat{q}_{z_{\mathrm{sb}}}}{q + \check{q}_{z_{\mathrm{sb}}}}\right), \tag{2}$$

with positive parameters $X_{z_{\mathrm{sb}}}^{\infty}$, $\hat{q}_{z_{\mathrm{sb}}}$ and $\check{q}_{z_{\mathrm{sb}}}$. For example, a sludge blanket at $z_{\mathrm{sb}} = 1$ m gives the function $U_1(q)$, whose graph is the dashed curve in Figure 1 (right).

Results and Discussion

For $w > 0$ we have shown (details are omitted here) that the steady states of the ASP, operating with a settler in normal operation, are given by a one-parameter family of solutions. The five unknowns S^*, X^*, X_{r}, r and w satisfy the following four equations (except for the trivial one $X_{\mathrm{e}} = 0$):

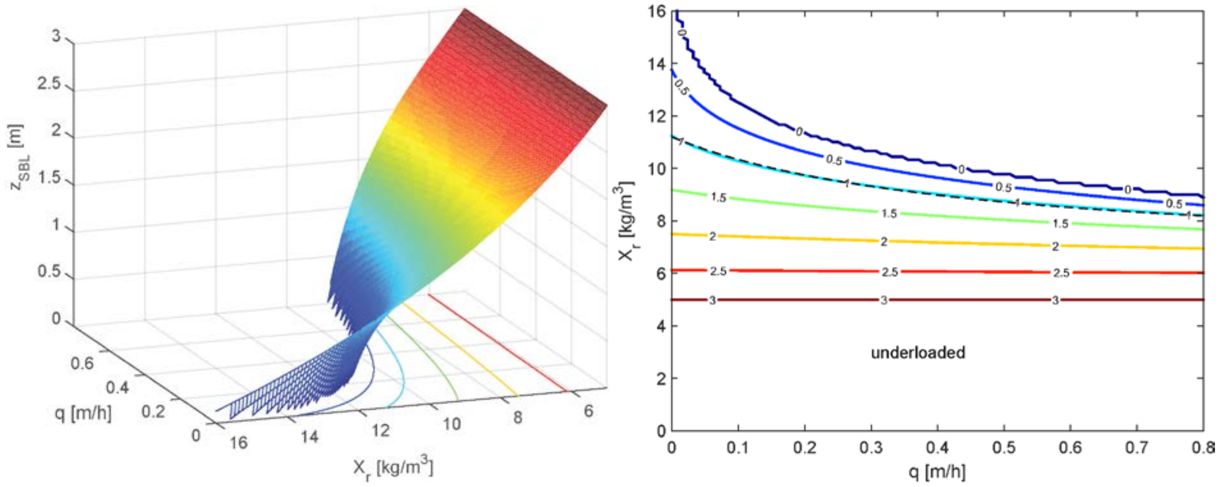

Figure 1. The sludge blanket level $z_{sb} = z_{sb}(q, X_r)$ for a settler in normal operation obtained from steady-state solutions of the PDE model. The graph and the contours of the function are shown in the left and right plot, respectively. The dashed curve in the right plot is the graph of the estimated funciton $X_r = U_1(q)$.

$$S^* = S^*(r,w) := S_{in} - \left(w + \frac{(r+w)Vb}{(1+r)Q}\right)\frac{U_{z_{sb}}(q(r,w))}{Y}, \tag{3}$$

$$X^* = \frac{r+w}{1+r}U_{z_{sb}}\big(q(r,w)\big), \tag{4}$$

$$X_r = U_{z_{sb}}\big(q(r,w)\big), \tag{5}$$

$$\theta(r,w)\big(\mu(S^*(r,w)) - b\big) = 1. \tag{6}$$

In (6), we have used that the sludge age θ can be written as a function of r and w:

$$\theta = \frac{VX^*}{wQX_r} = \frac{(r+w)V}{w(1+r)Q} =: \theta(r,w),$$

where we have used (4) and (5) to show that the sludge rate does not depend on any concentration variable.

We have proved that (for realistic parameter values) Eq. (6) defines implicitly w as a function of r, i.e. $w = w(r)$. This function can be substituted into (3)–(5) to obtain the concentrations as functions of r. Numerical solutions will be shown and discussed at the conference for different values on the decay rate b and the input variables S_{in} and Q.

A second outcome of the work is related to the fact that (3)–(6) contain the horizontal cross-sectional areas of the reactor A_R and settler A_S. The bioreactor volume can namely be written $V = A_R H_R$, where H_R is its height, and A_S can be found in q, see (1). Based on an analysis of the steady-state equations, we suggest the following design procedure.

Procedure for the design of an ASP

1. Determine all parameters of the biochemical and physical processes, i.e., Y, K_S, b and the parameters of the functions for the hindered settling velocity and the compression.
2. Define the depth B of the thickening zone of the settler and a suitable depth of the sludge blanket level $z_{sb} < B$. Compute the parameters $X^\infty_{z_{sb}}$, $\hat{q}_{z_{sb}}$ and $\check{q}_{z_{sb}}$ of the function in (2) (formulas are not given here).
3. Determine a reference value of the effluent substrate concentration $S^*_{ref} > K_s b/(\mu_{max} - b)$ and compute the corresponding sludge age $\theta_{ref} = 1/(\mu(S^*_{ref}) - b)$.

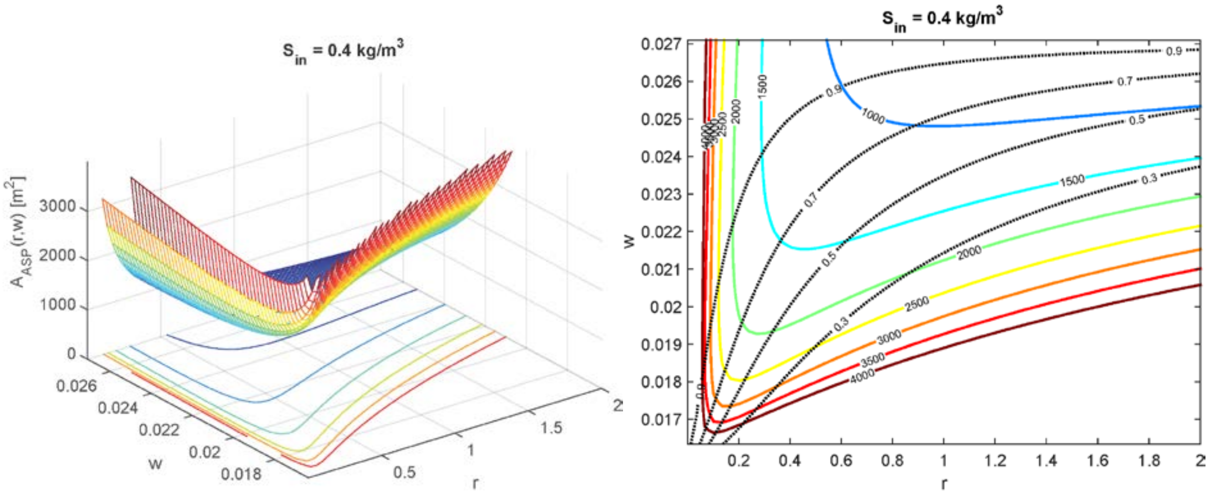

Figure 2. The graph and contours of $A_{\mathrm{ASP}}(r,w)$ showed in colours. The dotted black curves in the right plot show some ratios $A_{\mathrm{R}}/A_{\mathrm{ASP}}$. The w-axis shows the interval $w_{\min} < w < w_{\max}$, which corresponds to $X_{\mathrm{r,max}} = 11.20 > X_{\mathrm{r}} > X_{\mathrm{r,min}} = 6.52\ \mathrm{kg/m^3}$.

4. Choose a suitable depth of the bioreactor H_{R} and set Q and S_{in} to the maximum (steady-state) values which the ASP should be able to handle.
5. Investigate the possible values of the total area of the ASP; see Figure 2 for an example (formulas for $w_{\min}(S_{\mathrm{in}})$ and $w_{\max}(S_{\mathrm{in}})$ are not given here):

$$A_{\mathrm{ASP}}(r,w) := A_{\mathrm{R}}(r,w) + A_{\mathrm{S}}(r,w) = \frac{Qw(1+r)\theta_{\mathrm{ref}}}{(r+w)H_{\mathrm{R}}} + \frac{Q(r+w)(w_{\max}(S_{\mathrm{in}}) - w)}{(\hat{q}_{z_{\mathrm{sb}}} + \check{q}_{z_{\mathrm{sb}}})(w - w_{\min}(S_{\mathrm{in}}))},$$
$$r > 0,\ w_{\min}(S_{\mathrm{in}}) < w < w_{\max}(S_{\mathrm{in}})\,.$$

6. Choose first a required total area A_{ASP} (smaller means less start-up cost), then the ratio $A_{\mathrm{R}}/A_{\mathrm{ASP}}$, and finally an operating point (r,w) (smaller r means less running cost; energy for recirculating water). In the right plot of Figure 2, some curves of constant ratios $A_{\mathrm{R}}/A_{\mathrm{ASP}}$ are also shown. One choice of operating point can be the intersection of the green curve with $A_{\mathrm{ASP}} = 2500\ \mathrm{m}^2$ and the dotted black with $A_{\mathrm{R}}/A_{\mathrm{ASP}} = 70\%$. This gives the operating point $(r,w) = (0.213, 0.0196)$.

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Presenting Author

Dr Stefan Diehl
Lund University, Sweden

Is the presenting author an IWA Young Water Professional? N

Bio: Applied mathematician, who has spent too much time on administrative responsibilities at two universities, but sufficient time to develop a new type of batch-settling experiment and the Bürger-Diehl settler model. Interested in partial differential equations that arise in the modelling of wastewater treatment, mineral processing, chemical engineering and life science.