Optimizing zone volumes in bioreactors described by Monod and Contois growth kinetics

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Abstract: In this paper, an optimization study of simple bioreactors in series is presented. For a given total reactor volume, the zone volumes are optimized with respect to the effluent substrate. Also simple activated sludge models are considered. Both Monod and Contois functions are used for describing the growth kinetics. It is shown that the optimal zone volumes are very different depending on the choice of growth kinetics. In particular, it is studied how the influent steady state substrate concentration influence the optimal configuration.

Keywords: Activated sludge process; bioreactors; Monod kinetics; Contois kinetics; modelling; optimization.

Introduction

Optimal design of bioreactors has been extensively studied during the several decades. An early contribution can be found from Aris (1961). Concerning optimization of reactor volumes the typical approach is to find the minimum total tank volume for a given choice of the effluent substrate concentration (Hill and Robinson (1989), Nelson and Holder (2009), Harmand and Dochain (2005)). In this paper, we consider a related problem, we assume that a total reactor volume is given and we then seek the zone volumes "*the golden volumes*", which minimises the steady state effluent substrate concentration. That is, how to slice the reactor volume in an optimal way or more formally: given a total reactor volume V and a number of zones N, find the *golden volumes* $V_i,\ i = 1,2,\dots,N$ so that S_N (effluent steady state substrate concentration) is minimized. Such an optimization procedure is of practical relevance when the reactor basin already exists but we want to find a zone configuration.

In this paper, we compare the optimal zone volumes for simple bioreactors having growth kinetics of the biomass described by a Monod and a Contois function, respectively. In particular, it is studied how the influent steady state substrate concentration influence the optimal configuration. Extensions to simple activated sludge processes (ASP) are also included (see also Scuras et al. (2001) for a related approach). It is shown that the optimal zone volumes are quite different depending on the choice of growth kinetics.

Material and Methods

Consider a system of N simple bioreactors in series. The influent volumetric flow rate is equal to the effluent flow rate Q. The influent to the first reactor has a substrate concentration S_{in} and biomass X_{in}. It is assumed that no biomass is present in the influent. The differential equations describing the dynamics of the biomass and substrate in the i^{th} bioreactor are given by:

$$\begin{aligned} \dot{X}_i &= \mu(S_i, X_i)X_i + \frac{Q}{V_i}(X_{i-1} - X_i); \\ \dot{S}_i &= -\frac{\mu(S_i,X_i)}{Y}X_i + \frac{Q}{V_i}(S_{i-1} - S_i); \end{aligned} \qquad (1)$$

where S denotes substrate concentration, X denotes the biomass concentration, Y is the yield factor and μ is the specific growth rate. Subscript i refers to tank i.

The specific growth rate of the biomass for Monod and Contois kinetics are modelled by (2) and (3), respectively.

$$\mu(S,X) = \mu_M(S) = \mu_{max}\frac{S}{K_s+S} \quad (2)$$

$$\mu(S,X) = \mu_C(S,X) = \mu_{max}\frac{S}{XK_s+S} \quad (3)$$

where μ_{max} is the maximum specific growth rate and K_s is the half saturation constant. A detailed solution to (1) for Monod and Contois kinetics can be found in Carlsson and Zambrano (2014), and in Nelson and Holder (2009). In particular, the conditions for V_1 to avoid wash-out ($\bar{X} > 0$) are as follows:

$$V_1 > \frac{Q}{\mu_{max}}\left(\frac{K_s}{S_{in}} + 1\right) = \hat{V}_{1,M} \quad \text{(for Monod)} \quad (4)$$

$$V_1 > \frac{Q}{\mu_{max}} = \hat{V}_{1,C} \quad \text{(for Contois)} \quad (5)$$

where $\hat{V}_{1,M}$ and $\hat{V}_{1,C}$ are the minimum volumes to avoid wash-out both for Monod and Contois kinetics, respectively.

Optimization of volumes

Given a total reactor volume V and a number of zones N, find the optimal values of the golden volumes $(V_1^*, \dots, V_N^*)$ so that the effluent steady state substrate concentration ($\bar{S}_N$) is minimized. The problem can be expressed as:

$$\min_{(V_1^*,\dots,V_i^*,\dots,V_N^*)} \{\bar{S}_N\}; \quad (6)$$

subjected to:

$$\sum_{i=1}^{N} V_i^* = V; \;\; V_1^* > \hat{V}_1; \;\; V_i^* > 0; \quad i = 1,2,\dots,N \quad (7)$$

Next, consider the case of an ASP with N bioreactors in series. Let the model of the settler be defined by:

$$\begin{gathered}(Q+Q_r)X_N = (Q_r+Q_w)X_r;\\ S_N = S_r = S_{\text{eff}};\\ X_{\text{eff}} = 0;\end{gathered} \quad (8)$$

where Q_r and Q_w are the return sludge flow rate and the excess sludge flow rate, respectively. S_r and X_r are the substrate and the biomass concentration in the return sludge, respectively. S_{eff} and X_{eff} are the substrate and the biomass concentration in the effluent of the settler, respectively.

Here the optimization of the volumes for the bioreactors is identical to that defined by (6). The conditions for an ASP with one bioreactor to avoid wash-out ($\bar{X} > 0$) are as follows:

$$V_1 > \frac{Q_w}{\mu_{max}}\left(\frac{Q+Q_r}{Q_w+Q_r}\right)\left(\frac{K_s}{S_{in}} + 1\right) \quad \text{(for Monod)} \quad (9)$$

$$V_1 > \frac{Q_w}{\mu_{max}}\left(\frac{Q+Q_r}{Q_w+Q_r}\right) \quad \text{(for Contois)} \quad (10)$$

Results and Conclusions

In this section some numerical illustrations are shown. Let $V = 3$; $Q = 1$; $\mu_{max} = 2$; $Y = 0.8$; $K_s = 1.2$; $Q_r = 0.5$; $Q_w = 0.5$.

Illustrative example 1: Simple bioreactors

For the case of two bioreactors, Figure 1 shows the evolution of $\bar{S}_2$ for different values of the volume in the first bioreactor (V_1), both for Monod and Contois kinetics.

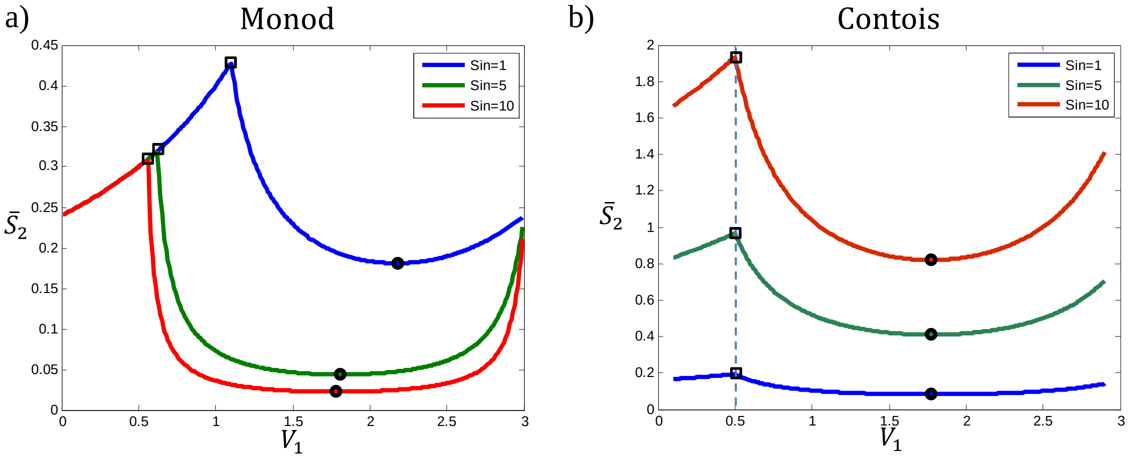

Figure 1. Evolution of $\bar{S}_2$ for different values of V_1. a) For Monod kinetics; b) For Contois kinetics. V_1^* are shown with a black circle. $\hat{V}_1$ are shown with a black square.

Note that for Monod, the golden volume V_1^* decreases as S_{in} increases, as well as the effluent decreases as the influent increases. For Contois, V_1^* is independent of S_{in} and the effluent value is proportional to the influent. Also note that $\hat{V}_{1,M}$ decreases as S_{in} increases, as seen in (4). On the other hand, $\hat{V}_{1,C}$ is independent of S_{in}, as seen in (5).

Then, we consider the problem of finding the golden volumes for more than two bioreactors. Table 1 shows the results for the golden volumes V_N^* both for Monod and Contois kinetics. Results are obtained by setting $S_{in} = 10$ and $X_{in} = 0$.

Table 1. Golden volumes for various bioreactors

	Monod						Contois					
N	V_1^*	V_2^*	V_3^*	V_4^*	V_5^*	$\bar{S}_N$	V_1^*	V_2^*	V_3^*	V_4^*	V_5^*	$\bar{S}_N$
1	3					2.4·10-1	3					1.61
2	1.78	1.22				2.2·10-2	1.7723	1.2277				8.2·10-1
3	1.40	0.82	0.78			3.3·10-3	1.42	0.83	0.75			5.5·10-1
4	1.19	0.63	0.60	0.58		6.9·10-4	1.26	0.65	0.56	0.53		4.3·10-1
5	1.096	0.508	0.476	0.464	0.456	1.7·10-4	1.18	0.54	0.45	0.42	0.41	3.6·10-1

Note that the effluent steady state substrate concentration decreases as the number of bioreactors increases.

Illustrative example 2: Activated Sludge Process

Consider the case of an ASP with two bioreactors, the evolution of $\bar{S}_{\mathrm{eff}}$ for different values of the volume in the first bioreactor (V_1), is depicted in Figure 2. Similar to the case of simple bioreactors, note that for Monod, the golden volume V_1^* decreases as S_{in} increases. For Contois, V_1^* is independent of S_{in}.

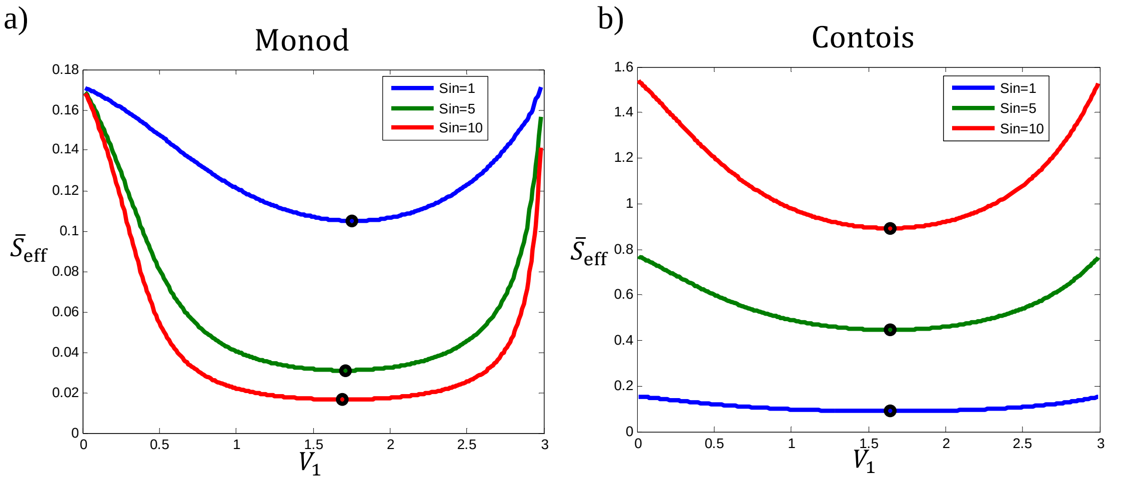

Figure 2. Evolution of $\bar{S}_{\text{eff}}$ for different values of V_1. a) For Monod kinetics; b) For Contois kinetics. V_1^* are shown with a black circle.

Then, we consider the problem of finding the golden volumes for more than two bioreactors in an ASP. Table 2 shows the results for the golden volumes V_N^* both for Monod and Contois kinetics. Results are obtained by setting $S_{in} = 10$ and $X_{in} = 0$.

Table 2. Golden volumes for an ASP

	Monod					Contois				
N	V_1^*	V_2^*	V_3^*	V_4^*	$\bar{S}_{\text{eff}}$	V_1^*	V_2^*	V_3^*	V_4^*	$\bar{S}_{\text{eff}}$
1	3				1.7·10-1	3				1.546
2	1.69	1.31			1.6·10-2	1.637	1.363			8.9·10-1
3	1.26	0.89	0.85		2.5·10-3	1.176	0.944	0.88		6.5·10-1
4	1.05	0.68	0.64	0.63	5.4·10-4	0.95	0.74	0.67	0.64	5.4·10-1

In this paper, the optimization of zone volumes for a given total volume bioreactor was studied. The zone volumes are optimized to minimize the effluent substrate. Both Monod and Contois kinetics were studied in two scenarios: simple bioreactors and activated sludge process. It is shown that the optimal zone volumes are quite different depending on the choice of growth kinetics.

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