Steady-state analysis of simple activated sludge processes with Monod and Contois growth kinetics

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Abstract: In this paper, simple activated sludge process (ASP) models with Monod and Contois biomass growth kinetics are compared. In particular, the cases of an ASP with one or two bioreactors are studied. Of particular concern in this study is to investigate how the effluent substrate concentration depends on the influent substrate concentration. It is shown that the results are very different depending on the number of bioreactors and on the growth kinetics assumed.

Keywords: Activated sludge process; bioreactors; sludge retention time; Monod; Contois; modelling.

Introduction

Mathematical models for the activated sludge process (ASP) have been an active research area for at least 50 years (and counting), see for example Henze *et al.* (1997) and the references therein. In this paper, we compare the steady state behaviour of an ASP with one and two bioreactors and with Monod and Contois growth kinetics with respect to how the effluent substrate concentration depends on the influent substrate concentration. To the best of the authors knowledge such a study is novel and increases the understanding of basic ASP models.

Material and Methods

Consider an ASP with N bioreactors in series, as shown in Figure 1, where the influent volumetric flow rate in every bioreactor is equal to its effluent flow rate. The substrate concentration is denoted S and the biomass concentration is X. The substrate is assumed to be soluble and its concentration is not affected by the settler, i.e $S_N = S_e = S_r$. No biomass is present in the influent, that is $X_{in} = 0$. No biological reactions take place in the settler and the settling is exhaustive, meaning that $X_e = 0$.

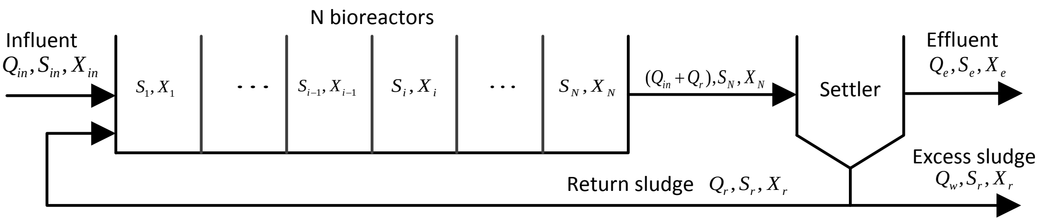

Figure 1. Activated sludge process with *N* bioreactors.

In steady state condition, the mass balances describing the biomass and substrate in the first reactor gives:

$$0 = \mu(S_1, X_1)X_1 + \frac{Q_r}{V_1}X_N - \frac{(Q_{in}+Q_r)}{V_1}X_1 \tag{1a}$$

$$0 = -\frac{\mu(S_1,X_1)}{Y}X_1 + \frac{Q_{in}}{V_1}(S_{in} - S_1) + \frac{Q_r}{V_1}(S_N - S_1) \tag{1b}$$

For the i^{th} ($i>1$) bioreactor gives:

$$0 = \mu(S_i, X_i)X_i + \frac{(Q_{in}+Q_r)}{V_i}(X_{i-1} - X_i) \tag{2a}$$

$$0 = -\frac{\mu(S_i,X_i)}{Y}X_i + \frac{(Q_{in}+Q_r)}{V_i}(S_{i-1} - S_i) \tag{2b}$$

where V_i is the volume of the i^{th} bioreactor, Y is the yield factor and μ is the specific growth rate (see also Figure 1 for definition of flow rates). In steady state, the mass balance for the settler (using the assumptions above) is:

$$(Q_{in} + Q_r)X_N = (Q_r + Q_w)X_r \quad (3)$$

The specific growth rate of biomass for Monod and Contois kinetics is given by:

$$\mu(S_i, X_i) = \mu_M(S_i) = \frac{\mu_{max} S_i}{K_s + S_i} \quad \text{(Monod)} \quad (4)$$
$$\mu(S_i, X_i) = \mu_C(S_i, X_i) = \frac{\mu_{max} S_i}{X_i K_s + S_i} \quad \text{(Contois)} \quad (5)$$

where μ_{max} is the maximum specific growth rate and K_s is a saturation constant.

ASP with one bioreactor – the classical text book scenario

For an ASP with one bioreactor (*N=1*), the mass balances are given by:

$$0 = \left[\mu(S_1, X_1) - \frac{1}{\theta_s}\right] X_1 \quad (6a)$$
$$0 = -\frac{\mu(S_1, X_1)}{Y} X_1 + \frac{Q_{in}}{V_1}(S_{in} - S_1) \quad (6b)$$

where θ_s is the sludge retention time (SRT), using (3) it gives:

$$\theta_s = \frac{V_1 X_1}{Q_w X_r} = \frac{V_1(Q_r + Q_w)}{Q_w(Q_{in} + Q_r)} \quad (7)$$

Note that using (3) in (7) no measurements of biomass concentrations are needed. The importance of the sludge age is that during steady state and no wash-out the following relation must hold:

$$\theta_s = \frac{1}{\mu(S_1, X_1)} \quad (8)$$

If more sludge is taken out from the system than is being produced, the result will be a *wash-out*, that is: $X_1 = 0$, $S_1 = S_{in}$. A condition for no wash-out can heuristically be derived by letting the SRT be larger than the SRT that would give $S_1 = S_{in}$, that is $\theta_s > 1/\mu(S_{in}, 0)$. For the case of Monod kinetics this gives:

$$\theta_s > \frac{1}{\mu_M(S_{in})} = \frac{K_s + S_{in}}{\mu_{max} S_{in}} \quad (9)$$

Using (4), (7) and (8), the effluent substrate concentration during no wash-out is:

$$S_1 = \frac{K_s}{\mu_{max}\theta_s - 1} = \frac{K_s Q_w(Q_{in} + Q_r)}{[\mu_{max} V_1(Q_r + Q_w)] - [Q_w(Q_{in} + Q_r)]} \quad (10)$$

In the case of Contois kinetics the condition for no wash-out is:

$$\theta_s > \frac{1}{\mu_C(S_{in}, 0)} = \frac{1}{\mu_{max}} \quad (11)$$

From (8), the effluent substrate concentration during no wash-out involves an expression for X_1. Note that (6a) and (6b) give:

$$X_1 = Y\theta_s \frac{Q_{in}}{V_1}(S_{in} - S_1) \quad (12)$$

Replacing (12) in (5), and then using (7) gives:

$$S_1 = \frac{Y K_s Q_{in} S_{in}}{\mu_{max} V_1 + Y K_s Q_{in} - \left(\frac{V_1}{\theta_s}\right)} = S_{in}\left\{\frac{Y K_s Q_{in}(Q_r+Q_w)}{[(Q_r+Q_w)(Y K_s Q_{in}+\mu_{max} V_1)]-[Q_w(Q_{in}+Q_r)]}\right\} \quad (13)$$

Three major differences between Monod and Contois kinetics are:

i. For Monod kinetics the wash-out condition depends on the influent substrate concentration, whereas for Contois kinetics it depends only on μ_{max}.

ii. For Monod kinetics the effluent substrate concentration (during no wash-out) is uniquely determined by the SRT and hence does not depend on the influent substrate concentration. See also Example 6.1 in Henze et al (1997). For Contois kinetics, the effluent substrate concentration is proportional to the influent substrate concentration.

iii. For Monod kinetics the effluent substrate concentration will decrease towards zero as the sludge age increases, whereas for Contois kinetics it will not.

ASP with multiple bioreactors

For more than one bioreactor, the SRT can be expressed by:

$$\theta_s = \frac{\sum_{i=1}^{N} V_i X_i}{Q_w X_r} \quad (14)$$

In the same way as in the case of an ASP with one reactor, a condition for no wash-out can be heuristically derived by letting the SRT be larger than the SRT that would give $S_1 = S_{in}$ (and $X_1 = 0$), that is $\theta_s > 1/\mu(S_{in,}0)$. It does not seem possible to find explicit solutions for S_N and X_N for the case $N \geq 2$. In this paper we therefore illustrate the case *N=2* by a numerical study. In contrast to the single bioreactor case, we will show that for a given sludge age the effluent substrate depends on the influent substrate. Analytical solutions for simplified reactor configurations with Contois kinetics have recently been presented by Alqahtani *et al.* (2012, 2013).

Numerical results

ASP with one and two bioreactors is taken as case study. Here (assuming appropriate units), $V = 9$; $Q_{in} = 1$; $Q_r = 2$; $\mu_{max} = 0.1$; $Y = 0.8$; $K_s = 2$; $S_{in} = [10, 20, 30]$. Changes in θ_s were done by adjusting Q_w ($0 \leq Q_w \leq Q_{in}$).

Illustrative example 1: ASP with one bioreactor

Figure 2 shows the steady state value of S_1 as a function of θ_s for Monod and Contois kinetics. It gives an illustration of items (i) and (ii) above.

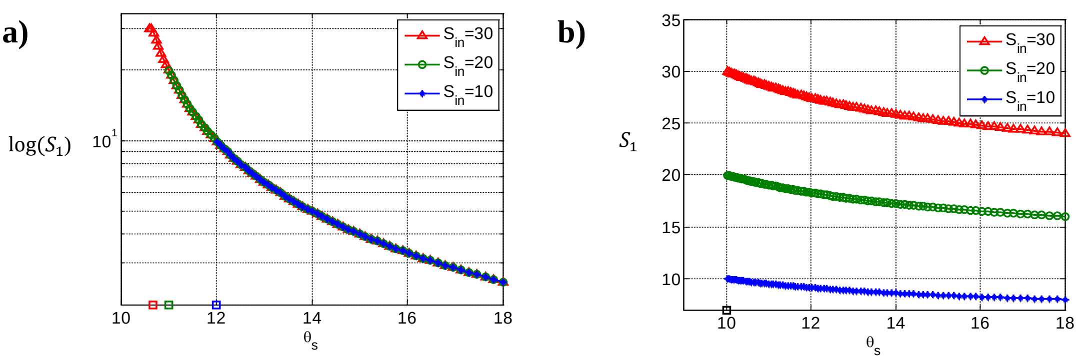

Figure 2. S_1 as a function of θ_s for different S_{in}. **a)** Monod kinetics. **b)** Contois kinetics. Squares in the θ_s-axis denote minimum values of θ_s to avoid wash-out.

Illustrative example 2: ASP with two bioreactors

In this case $V_1 = V_2 = V/2$ is chosen. Figure 3 shows the steady state value of S_2 as a function of θ_s. Results are shown for Monod and Contois kinetics.

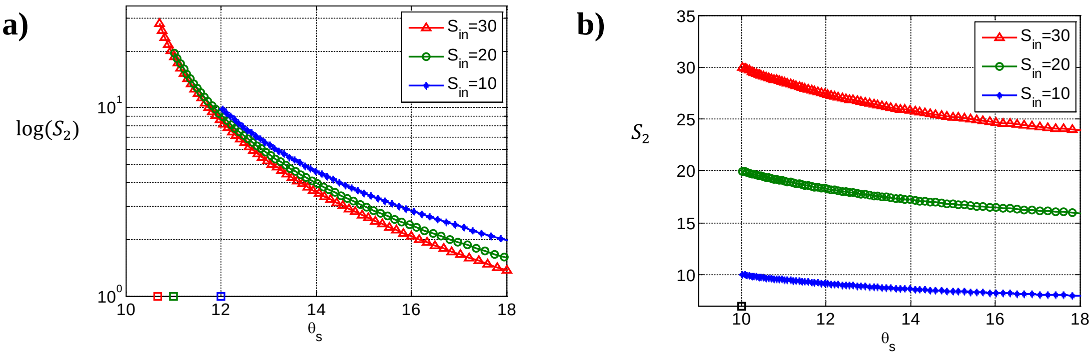

Figure 3. S_2 as a function of θ_s for different S_{in}. **a)** Monod kinetics. **b)** Contois kinetics. Squares in the θ_s-axis denote minimum values of θ_s to avoid wash-out.

The values of θ_s to avoid wash-out are numerically very similar to those obtained for the case of an ASP with one bioreactor. For Monod kinetics using a fixed sludge age, the effluent substrate decreases as the influent substrate increases. The Contois kinetics case gives a similar behaviour for one and two bioreactors.

Conclusions

In this paper, we have compared simple ASP's models when the specific growth rate is described by Monod and Contois kinetics. For the case of one bioreactor the comparison were made analytically and for the case of two bioreactors a numerical study was used. In summary, it is shown that the behaviour of a basic ASP model can be very different if Monod or Contois kinetics are used, and also if the ASP has one or multiple bioreactors. For example, the classical result for an ASP with one bioreactor and Monod kinetics that the effluent substrate concentration depends only of the sludge age does not hold when using multiple bioreactors, the effluent concentration then decreases as the influent substrate increases.

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