Analysis of simple bioreactor models - a comparison between Monod and Contois kinetics

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Abstract: In this paper, an analysis of simple bioreactors in series is presented. The bioreactors are analysed for growth kinetics of the biomass described by a Monod and a Contois function. In particular, it is studied how the effluent substrate concentration is depending on the influent substrate concentration during steady state. It is shown that by going from one to two bioreactors in series completely changes the process behaviour when the growth kinetics is described by a Monod function. It is also shown that a bioreactor described by Contois kinetics has a completely different behaviour compared to the Monod case.

Keywords: Bioreactors; activated sludge process; Monod kinetics; Contois kinetics; modelling.

Introduction

The heart of an activated sludge process (ASP) is the biological reactor. It is hence of fundamental importance to understand the processes going on in the bioreactor. Numerous text books and research papers are dealing with modelling, analyses and optimization of bioreactors. However, to the best of the authors knowledge, an analyses on how the effluent substrate concentration is affected by the influent substrate concentration during steady state has not been completely covered, and in particular comparing this behaviour on bioreactors with growth kinetics of the biomass described by a Monod and a Contois function, respectively.

Consider a simple (only one limiting substrate and one type of biomass) continuous stirred tank reactor (CSTR) having at least two bioreactors in series and a growth kinetics of the biomass described by a Monod function. In this paper, we will show analytically that the effluent steady state concentration of substrate decreases when the influent concentration of substrate increases. This behaviour is very different from the classical one bioreactor case, where the effluent steady state substrate concentration is independent of the influent substrate concentration. We will also show that if the growth kinetics is described by a Contois function a completely different behaviour is obtained. Then the effluent substrate concentration (in steady state) is proportional to the influent substrate concentration. The implication of the above results for an ASP is outlined in a companion paper Zambrano and Carlsson (2014) where it numerically is shown that for a simple ASP process described by two bioreactors in series the same behaviour is obtained.

Microbial growth in CSTRs

We consider the dynamic of a single CSTR. The differential equation describing the dynamics of the biomass and substrate is given by (see, for example, Dochain and Vanrolleghem (2001)):

$$\frac{dX}{dt} = \left[\mu(S,X) - \frac{Q}{V}\right] X + \frac{Q}{V} X_{in} \quad (1)$$

$$\frac{dS}{dt} = -\frac{\mu(S,X)}{Y} X + \frac{Q}{V}(S_{in} - S) \quad (2)$$

where S and X are the substrate and biomass concentration in the bioreactor, respectively. The influent volumetric flow rate is equal to the effluent flow rate Q. S_{in}

and X_{in} are the influent substrate and biomass concentration, respectively. V is the bioreactor volume, Y is the yield factor and $\mu(S,X)$ is the specific growth rate. It will generally be assumed that (1) and (2) have the initial conditions $X(0) > 0$ and $S(0) \geq 0$, respectively. In steady state ($0 = dX/dt = dS/dt$) we have:

$$\bar{X} = Y(S_{in} - \bar{S}) + X_{in} \tag{3}$$

where the steady state points are denoted by $\bar{X}$ and $\bar{S}$.

A single CSTR with $X_{in} = 0$

This is the classical set-up frequently covered in text books like Smith and Waltman (1995). For completeness we outline the results. Let first the growth kinetics be described by a Monod function:

$$\mu(S,X) = \mu_M(S) = \frac{\mu_{max}S}{K_s+S} \tag{4}$$

where μ_{max} is the maximum specific growth rate and K_s is the half saturation constant. The conditions[1] to avoid wash-out ($\bar{X} > 0$) are given by:

$$S_{in} > \frac{QK_s}{V\mu_{max}-Q} = \bar{S}, \qquad \mu_{max} > \frac{Q}{V} \tag{5}$$

In this case, $\bar{S}$ does not depend on S_{in}.

For the Contois kinetics, the specific growth rate of the biomass is modelled by:

$$\mu(S,X) = \mu_C(S,X) = \frac{\mu_{max}S}{XK_s+S} \tag{6}$$

The solution for $\bar{S}$ during non-wash-out condition is:

$$\bar{S} = S_{in}\left(\frac{QK_sY}{V\mu_{max}+QK_sY-Q}\right) \tag{7}$$

Note that $\bar{S}$ is proportional to S_{in}. A necessary condition for a CSTR with Contois kinetics to avoid wash-out is:

$$\mu_{max} > \frac{Q}{V} \tag{8}$$

A single CSTR with $X_{in} > 0$

This is the general case for a single CSTR. For Monod kinetics the expression for $\bar{S}$ can be obtained by inserting (3) in (2) and using (4):

$$\bar{S}^2 - (A + S_{in} + B)\bar{S} + S_{in}B = 0 \tag{9}$$

where $A = \frac{V\mu_{max}X_{in}}{Y(V\mu_{max}-Q)}$ and $B = \frac{QK_s}{(V\mu_{max}-Q)}$. Inserting (3) in (2) and using (6) gives for the Contois kinetics:

$$\bar{S}^2 - (A + S_{in} + \bar{X}B)\bar{S} + S_{in}B\bar{X} = 0 \tag{10}$$

[1] A formal mathematical proof using Lyapunov stability analyses can be found in Rao and Rao (2009).

A detailed solution to (9) and (10) can be found in Carlsson and Zambrano (2014), and in Nelson and Holder (2009), respectively.

Two CSTRs in series

In this section, a system of two CSTRs in series is considered. The influent to the first bioreactor has a substrate concentration S_{in} but no biomass X_{in}=0 and, for simplicity, it is assumed that both bioreactors have the same volume V. Proceeding as in the single CSTR case gives:

$$\begin{aligned} \frac{dX_1}{dt} &= \left(\mu_1 - \frac{Q}{V}\right) X_1; & \frac{dX_2}{dt} &= \mu_2 X_2 + \frac{Q}{V}(X_1 - X_2); \\ \frac{dS_1}{dt} &= -\frac{\mu_1}{Y} X_1 + \frac{Q}{V}(S_{in} - S_1); & \frac{dS_2}{dt} &= -\frac{\mu_2}{Y} X_2 + \frac{Q}{V}(S_1 - S_2); \end{aligned} \tag{11}$$

It is the first bioreactor volume that determines the wash-out condition. Hence, exactly as for the single CSTR case, the wash-out conditions for two CSTRs in series are given by (5) for Monod and by (8) for Contois.

For Monod kinetics, by replacing (3) and (5) in (9), the solution for $\overline{S_2}$ satisfies the quadratic equation:

$$\overline{S_2}^2 - \overline{S_2}\left[\frac{V\mu_{max}(S_{in}-\overline{S_1})+2QK_s}{V\mu_{max}-Q}\right] + \left(\frac{QK_s}{V\mu_{max}-Q}\right)^2 = 0 \tag{12}$$

It is clearly seen that in this case $\overline{S_2}$ depends on S_{in}. It is straightforward to show (Carlsson and Zambrano (2014)) that:

$$\frac{\partial \overline{S_2}}{\partial S_{in}} < 0$$

and

$$\overline{S_2} \to 0 \quad \text{as} \quad S_{in} \to \infty$$

Hence, in contrast to the single basin case, *the effluent substrate decreases as the influent substrate increases* and in the limit goes to zero. Intuitively, this can be explained as follows. The substrate concentration from the first bioreactor is constant, see (5), but the biomass will increase with S_{in}, see (3) with X_{in}=0. The influent to the second bioreactor will hence have a constant substrate load but a biomass concentration that increases with S_{in}. In the limit, the biomass concentration goes to infinity and will consume all substrate in the second bioreactor.

For Contois kinetics, by replacing (3) and (7) in (10), the solution for $\overline{S_2}$ gives:

$$\overline{S_2} = \frac{S_{in}}{k_7}[k_1 + k_2 + k_3 - k_4 - k_5 - k_6(\mu_{max}V - Q)] \tag{13}$$

where $k_1 = 2(K_sQY)^2$; $k_2 = (\mu_{max}V)^2$; $k_3 = 2K_s\mu_{max}QVY$; $k_4 = 2K_sQ^2Y$; $k_5 = \mu_{max}QV$; $k_6 = \sqrt{\mu_{max}V[2(k_1 + k_3 - k_4) + k_2 - k_5]/(\mu_{max}V - Q)}$; and $k_7 = 2\left(\frac{k_1}{2} + k_2 + k_3 - k_4 - 2k_5 + Q^2\right)$. Note that (13) shows a linear dependence of $\overline{S_2}$ with respect to S_{in}.

Numerical illustration

As a numerical illustration, a single CSTR and two CSTRs in series are taken as case study. Here (assuming appropriate units), $Q = 1$; $V = 3$; $X_{in} = 0$; $\mu_{max} = 2$; $Y = 0.8$; $K_s = 1.2$. For one CSTR, $V = V_1$ and for the two CSTRs case, $V_1 = V_2 = V/2$ is used. Figure 1 shows the effluent steady state concentration as a function of the influent S_{in}, considering Monod and Contois kinetics.

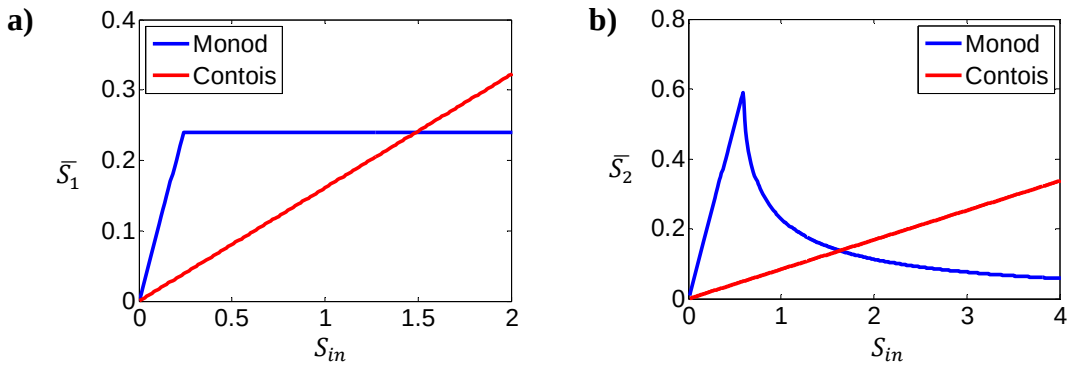

Figure 1. Steady state concentration of the effluent substrate as a function of S_{in}. **a)** One CSTR; **b)** Two CSTRs in series. Monod (blue) and Contois (red) kinetics.

First consider the Monod kinetics. For the case of one CSTR wash-out is avoided when $S_{in} > \frac{QK_s}{V\mu_{max}-Q} = 0.24$. During no wash-out the effluent substrate concentration does not depend on S_{in}, as supported by (5). For two CSTRs, the wash-out is avoided when $S_{in} > \frac{QK_s}{(V/2)\mu_{max}-Q} = 0.6$. During no wash-out condition, the effluent substrate concentration decreases as S_{in} increases, as supported by the solution of (12). For the Contois kinetics, the effluent substrate concentration has a linear dependence with respect to S_{in}, both for the case of a single CSTR and two CSTRs in series, as supported by (7) and (13).

Conclusions

We have shown (both analytically and by simulation studies) that a bioreactor model can give very different results depending on the bioreactor configuration (one or multiple zones) and on the choice of growth kinetics (Monod or Contois), see Figure 1. We believe that the results bring some new insight in understanding basic bioreactor models including its application to activated sludge process models.

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